Endogenous risk

Dual Role of Prices

1987

Actual and perceived risk

LTCM

Financial Risk Forecasting
Chapter 10
Endogenous Risk

Jon Danielsson ©2021
London School of Economics

To accompany
Financial Risk Forecasting
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<table>
<thead>
<tr>
<th>Endogenous risk</th>
<th>Dual Role of Prices</th>
<th>1987</th>
<th>Actual and perceived risk</th>
<th>LTCM</th>
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Endogenous risk

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LTCM
What is risk?

• Systemic risk and financial stability and economic growth
• Are directly dependent on risk
• The Goldilocks challenge
  • not too much and not too little, just right
• But then we have to know what risk is
Endogenous Risk (ER)
Butterflies and hurricanes

- Chaos theorists talk about how a butterfly in Hong Kong can cause a hurricane in the Caribbean
- What is important is the *mechanism* allowing this to happen
- The trigger (the butterfly) is incidental
- And the hurricane the unfortunate outcome
- Focus of study and policy should be the mechanism
Keynesian beauty contest

“It is not a case of choosing those [faces] which, to the best of ones judgement, are really the prettiest, nor even those which average opinion genuinely thinks the prettiest. We have reached the third degree where we devote our intelligences to anticipating what average opinion expects the average opinion to be. And there are some, I believe, who practice the fourth, fifth and higher degrees.”

Keynes, General Theory of Employment Interest and Money, 1936.
Endogenous risks vs. Exogenous risks

- **Endogenous risk**: the risk from shocks that are generated and amplified *within* the financial system
- **Exogenous risk**: shocks that arrive from *outside* the financial system
- Analogies
  - A financial hedge (futures contract) vs. a weather hedge (umbrella)
  - Poker vs. Roulette
- Essentially situations where an agent affects outcomes vs. situations where the agent cannot
Millennium Bridge

- First new Thames crossing for over a hundred years
  - New design, extensive tests, riskless
  - Opened by the Queen on June 10th 2000
- What happened?
  - Wobbled violently within moments of bridge opening
  - Remain closed for the next 18 months
Millennium Bridge

- New design
- Tested with extensive simulations
- All angles covered
- No endogenous shocks
- Riskless
What endogeneity?

- Pedestrians had some problems
- Bridge closed
What happened?

- Took the engineers some time to discover what happened
What went wrong?

- An engineering answer
  - Cause: horizontal vibrations at 1 hertz
  - Walking pace: 2 steps per second, i.e. 2 hertz
  - Producing 1 hertz horizontal force

- Why should it matter?
  - People swayed to the left and right cancel out each other
  - Only a problem when people walk in step like soldiers marching
  - Probability of a thousand people walking at random ending up walking exactly in step? — *close to zero*
  - If individual steps are independent events, but...
Given feedback...near certainty!

Bridge moves
Given feedback...near certainty!

Bridge moves → Adjust stance
Given feedback...near certainty!

Adjust stance

Bridge moves

Push bridge
Given feedback...near certainty!

- Adjust stance
- Bridge moves
- Push bridge
- Further adjust stance
Dual role of prices
Dual role of prices
Prices of financial assets play two important roles. The first is quite familiar

1. Prices reflect the underlying fundamentals
2. Prices are also an imperative for action
Endogenous market prices
Leverage constraints and upward sloping demand

- Leverage constraint $L = 5$ (assets to equity)
- Initial (time 0) values
  - Prices, $P_0 = $10
  - Number of assets, $Q_0 = 100$
  - Assets, $A_0 = $1000
  - Debt, $D_0 = $800
  - Equity, $E_0 = A_0 - D_0 = $200
- Leverage
  $$L = \frac{A}{A - D} = \frac{1000}{200} = 5$$
Prices fall to $P_1 = $9 at time 1

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
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<tbody>
<tr>
<td>1000</td>
<td>Equity 200</td>
</tr>
<tr>
<td></td>
<td>Debt 800</td>
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</table>

Leverage to 9, bank needs to *sell assets and repay debt*
Prices are exogenous

\[ L_1 = \frac{A_1}{A_1 - D_1} = \frac{P_1 Q_1}{P_1 Q_1 - (D_0 - P_1 (Q_0 - Q_1))} \]

so

\[ Q_1 = -L \frac{D_0 - P_1 Q_0}{P_1} \]

In our case \( Q_1 = \frac{500}{9} \), so the bank sells $400 worth of assets. Its balance sheet becomes:

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
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<tbody>
<tr>
<td>( A = 9 \frac{500}{9} = 500 )</td>
<td>( E = 500 - 400 = 100 )</td>
</tr>
<tr>
<td>( D = 800 - 9 \frac{400}{9} = 400 )</td>
<td>( D = 800 - 9 \frac{400}{9} = 400 )</td>
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</tbody>
</table>
Prices are endogenous
bank exerts significant price impact

\( \lambda \) is the *price impact factor*, and \( P_1 Q_1 \) the amount the bank wants to sell, in our case 400. Make \( \lambda = 0.001 \)

1. \( P_n = P_{n-1} + \lambda P_{n-1} (Q_{n-1} - Q_{n-2}) \);
2. \( Q_n = L (Q_{n-1} - D_{n-1}/P_n) \);
3. \( A_n = P_n Q_n \);
4. \( E_n = A_n / L \);
5. \( D_n = A_n - E_n \).
### Dual Role of Prices

<table>
<thead>
<tr>
<th>Iteration</th>
<th>$Q$</th>
<th>$P$</th>
<th>$A$</th>
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<td>100.000</td>
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### Iteration Table

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<td>Iteration</td>
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</table>
Demand — Upward sloping

Exogenous
Endogenous risk

Demand — Upward sloping

Exogenous

Endogenous
Butterflies and financial crises

• Demonstrates how a small exogenous shock can trigger a large outcome
• The constraints dictate a “sell cheap, buy dear” strategy
• Precisely the kind of vicious feedback loops that destabilize markets
• This is the mechanism that allows the butterfly to create the hurricane
Dynamic Strategies and the ’87 Crash
Black Monday

- October 19, 1987 — the biggest stock market crash since the 19th century
- Global stock markets crashed around 23%
- A key reason for the crash was *portfolio insurance*
- That is, the use of an automatic trading strategy
Endogenous risk

Dual Role of Prices

1987

Actual and perceived risk

LTCM

1987 Dow Jones Industrial Average index values
Put option and Delta

• A **put option** gives the holder the right to sell an asset at an agreed **strike price** ($X$)

• **Delta** ($\Delta$) of a put option is the rate of change of its price ($\Pi$) with respect to the change in price of the underlying asset, $P$

$$\Delta = \frac{d\Pi}{dP} < 0$$

• Graphically, $\Delta$ is the slope of a curve — the option price against the price of the underlying
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Dual Role of Prices

Payoff, or \( \Pi \), at expiration

Put option and Delta
Dual Role of Prices

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Put option and Delta

Payoff, or $\Pi$, prior to expiration

Payoff, or $\Pi$, at expiration
Dual Role of Prices

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Put option and Delta
Hedging through options

- Options are effective and instruments to hedge
- Delta hedging
  - A portfolio that consists of 1 put option and $\Delta$ stocks is riskless
  - and must earn the risk–free rate
- Traded options don’t always exist — long maturity, OTC markets
Synthetic options

- Financial engineering: turn one asset into another
- *Synthetic replication*: create an option by a combination of cash and the underlying asset
- An example — dynamically replicating a put

\[
\begin{align*}
1 \text{ put} &= \left\{ \begin{array}{l}
\Delta \quad \text{underlying asset} \\
0 \quad \text{cash}
\end{array} \right.
\\
0 \quad \text{cash} &= \left\{ \begin{array}{l}
-P\Delta + \Pi \\
\text{cash}
\end{array} \right.
\end{align*}
\]

- $\Delta$ of a put option is always negative
- Replicating portfolio: short $|\Delta|$ units of underlying asset at price $P$ at all times
Synthetic put

- Δ of the option becomes more negative as asset price falls
- "sell cheap, buy dear" strategy
Simulation

- Strike price at $90
- The risk–free rate at 0%
- The annual volatility at 25%
- Time to maturity is 9 weeks
- Initial price $100
- $\epsilon$ is shock
- Black–Sholes put is $0.8012$
- Use superscript * to denote the actual outcomes, so for example $P$ refers to the theoretic price and $P^*$ to the actual price
## Dynamic replication strategy

<table>
<thead>
<tr>
<th>$T - t$</th>
<th>$\epsilon$</th>
<th>$P$</th>
<th>$P^*$</th>
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<td>-1.00</td>
<td>-1.00</td>
<td>87.6</td>
<td>71.5</td>
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Dynamic replication strategy

Theoretical price

T−t

9/10 8/10 7/10 6/10 5/10 4/10 3/10 2/10 1/10 0/10
Dynamic replication strategy

Theoretical price
Actual Price

T–t

The graph shows the dynamic replication strategy over time, with the theoretical price and actual price plotted against time (T–t) from 9/10 to 0/10.
The crash

- $60–90 billion in formal portfolio insurance (3% of pre-crash market cap)
- Oct 14 (wed) to Oct 16 (Fri)
  - Market decline of 10%
  - Sales dictated by dynamic hedging, $12bn.
  - Actual sales (cash + futures), $4bn.
  - Substantial pent up selling pressure on Monday
Endogenous market dynamics

portfolio insurance

Price falls
Endogenous market dynamics

portfolio insurance

Price falls → Delta falls, sell

1987

Actual and perceived risk

LTCM
Endogenous market dynamics
portfolio insurance

Price falls → Delta falls, sell → Depress price
Endogenous market dynamics

portfolio insurance

- Price falls
- Delta falls, sell
- Depress price
- Delta falls more, sell
Trading rules

- Classic example of destabilizing feedback effect on market dynamics of concerted selling pressures arising from certain mechanical trading rules like the sell–on–loss considered here.
- The underlying destabilizing behavior is completely invisible so long as trading activity remains below some critical but unknown threshold.
- Only when this threshold is exceeded does the endogenous risk becomes apparent, causing a market crash.
- Clearly demonstrates the difference between perceived risk and actual risk.
Actual and Perceived Risk
A baby risk model

• Suppose returns are given by

\[ y_t = \frac{P_t - P_{t-1}}{P_{t-1}} \]

• Suppose we measure exogenous risk by volatility

\[ \text{volatility}_t^2 = \text{variance} = \frac{1}{N-1} \sum_{i=1}^{N} (y_{t-i} - \text{mean})^2 \]

• Measured risk is a function of historical prices

• Recall the example of the dam bursting in Chapter 1
“The received wisdom is that risk increases in recessions and falls in booms. In contrast, it may be more helpful to think of risk as increasing during upswings, as financial imbalances build up, and materialising in recessions.” Andrew Crockett, then head of the BIS, 2000

- Consistent with Minsky’s financial instability hypothesis
Relevance of endogenous risk

- When individuals observe *and* react — affecting their operating environment
- Financial system is not invariant under observation
- We cycle between virtuous and vicious feedbacks
- Two faces of risk
  - risk reported by most risk forecast models — perceived risk
  - actual underlying risk that is hidden but ever present
Endogenous bubble

Endogenous risk

Dual Role of Prices

1987

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Endogenous risk

Dual Role of Prices

1987

Actual and perceived risk

LTCM

Endogenous bubble

- Prices
- Perceived risk
Endogenous bubble

- Prices
- Perceived risk
- Actual risk
The 43 year cycle of systemic risk

actual risk builds up

2000 2010 2020 2030 2040
The 43 year cycle of systemic risk

actual risk builds up

2000 → 2010 → 2020 → 2030 → 2040

hidden trigger
The 43 year cycle of systemic risk

- Perceived risk indicators flash
- Actual risk builds up
- Hidden trigger

2000 → 2010 → 2020 → 2030 → 2040
The 43 year cycle of systemic risk

- Perceived risk indicators flash
- Actual risk builds up
- Improvised responses
- Hidden trigger

- 2000
- 2010
- 2020
- 2030
- 2040
The 43 year cycle of systemic risk

actual risk builds up

hidden trigger

perceived risk indicators flash

improvised responses

MacroPru implemented

2000 2010 2020 2030 2040
The 43 year cycle of systemic risk

perceived risk indicators flash

actual risk builds up

2000 2010 2020 2030 2040

improvised responses actual risk builds up

hidden trigger

MacroPru implemented
The 43 year cycle of systemic risk

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- MacroPru implemented

2000 → 2010 → 2020 → 2030 → 2040
The 43 year cycle of systemic risk

- Perceived risk indicators flash
- MacroPru implemented
- Improvised responses
- Hidden trigger

The 42 year cycle

2000 2010 2020 2030 2040

LTCM

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Actual and perceived risk
The 43 year cycle of systemic risk

perceived risk indicators flash

improvised responses

hidden trigger

MacroPru implemented

The 42 year cycle

Actual risk

Perceived risk

2000 2010 2020 2030 2040
The impact of managing risk

• Imagine two different worlds
  A. there are no regulations about risk and banks are all doing their own thing
  B. we have harmonized regulations and stakeholders that demand state of the art risk management

• Which world is more stable?
• It depends
• Because the management of risk controls only one part of the domain for risk
• Consider the day at the height of the Covid-19 market turmoil in the middle of March 2020. Would you expect perceived risk to increase over the subsequent few days? Would you expect actual risk to increase over the subsequent few days?
Impact of active risk management

Distribution of Risk

Market outcomes

without active risk management
Impact of active risk management

with active risk management

without active risk management

Market outcomes

Distribution of Risk

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Actual and perceived risk

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Impact of active risk management

Distribution of Risk

Market outcomes

with active risk management

without active risk management

Risk level targeted by active risk management

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Impact of active risk management

with active risk management

without active risk management

Risk of very large and uncommon outcomes

Risk level targeted by active risk management

Distribution of Risk

Market outcomes
The LTCM Crisis
Long–Term Capital Management

- Founded by John Meriwether, experts include Robert Merton and Myron Scholes
- Min investment $10mn, charge 2 and 25, 3 yrs lock in
- $1.01 billion in capital to start with
- Performance
  - First two years: 43% and 41% after fees
  - Net capital in September 1997 was $6.7 billion
  - Leveraged to $126.4 billion (19 times)
Long Short–Term Capital Management

- Founded by John Meriwether, experts include Robert Merton and Myron Scholes
- Min investment $10mn, charge 2 and 25, 3 yrs lock in
- $1.01 billion in capital to start with
- Performance
  - First two years: 43% and 41% after fees
  - Net capital in September 1997 was $6.7 billion
  - Leveraged to $126.4 billion (19 times)
- Failed spectacularly in 1998
Leverage

- “LTCM would make money by being a vacuum sucking up nickels that no one else could see.”
  - Myron Scholes
- Drove very hard bargains on financing
Trading strategies

- Convergence or relative value trades
- Examples:
  - Fixed rate the residential mortgages in US
  - Japanese and European government bonds
  - Interest rate swaps
  - Italy
The bigger picture

- VaR in 1998 indicated that it would take a $10\sigma$ event for it to lose all capital in a single year ($p = 10^{-24}$)
  - probability of default of $7.6 \times 10^{-23}$
  - the earth is $4.5 \times 10^9$ years old and the universe is $1.3 \times 10^{10}$ years old
- Returned $2.7bn.$ to investors in December 1997 (focus on investing own money)
- (And we worry about incentives in bonuses!)
- Copycat funds, proprietary trading desks of creditor banks → Narrowing of spreads
- Venturing into uncharted territory in search of profitable trades
“Central bank of volatility”

- The biggest trade in 1998 was to sell/write long-dated options
- Expect vol to go to long-run level
- LTCM became a major supplier of S&P 500 vega
- High leverage to profit from minute difference in value
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VIX

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LTCM

VIX

Long run mean = 17

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VIX — Close to the crime — 1998
Endogenous risk

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LTCM

VIX — Close to the crime — 1998
A perfect storm

- Returns: -6.7% May, -10.1% June 1998
- Leverage became 31/1 (capital drops relatively more than assets)
- Salomon Smith Barney closed US bond arbitrage group
- Russia default August 17th, triggered a panic
- Credit spreads widened, volatility shot up to 45% (the unthinkable happened)
- All correlations tended to 1 (as happens in crises)
Endogenous collapse

Margin calls
Endogenous collapse

- Margin calls
- Deleveraging
Endogenous collapse

- Deleveraging
  - Margin calls
  - Adverse price move
Endogenous collapse

- Margin calls
- Deleveraging
- Adverse price move
- Distress

- Endogenous risk
- Dual Role of Prices
- 1987
- Actual and perceived risk
- LTCM
Deleveraging

- Mutually reinforcing effect of deleveraging
- Distress and margin calls entails short–horizon trading
- Loosing more than $45 million per day
- In the first 3 weeks of Sept, equity tumbled from $2.3bn to $600mn
- Fed organized a $3.625bn rescue
Summary preconditions for endogenous risk

- Individual economic agents react to outcomes
- Individual actions affect outcomes
- To believe that LTCM was just hugely unlucky is to commit the same mistake as the engineers of the Millennium Bridge
- Far from a probability close to zero, the collapse was near certain given the right conditions
Irrationality of markets?

- LTCM invested in a mean-reverting asset, expecting VIX to eventually fall, bringing significant profits
- Profits were made, but only by those who bailed LTCM out
- The explanation is provided by an observation often attributed incorrectly to Keynes
  
  “The market can stay irrational longer than you can stay solvent.”

- The very high levels of VIX were explicitly caused by the uncertainty created by the presence of LTCM
- A necessary condition for the VIX to return to its long-run mean was the failure of LTCM